

MAIN ACTIVITIES

If you try to put more than one tile on a square on the eXpresser working area an “X” appears, because each square can only have one tile on it. When you create models in the eXpresser using more than one pattern, sometimes parts of the patterns overlap, like this:




If you want to remove a part of a pattern because it overlaps, you need to **create another pattern**, which is exactly the same as the tiles you want to get rid of, and then tell the computer to ‘**remove**’ that pattern from the model. There are several metaphors to help think of this. Patterns that are removed from the model are in essence negative numbers and so can be called **negative patterns**. Some students called these patterns ‘eraser’ patterns. In the dog model above, we would need to create a pattern made up of just the tail of the dog, and then think about how many tails need to be removed (how many tails are overlapping). The challenge in these occasions is to think how to do this in a general way. In both patterns above the number of tiles to be removed is one less than the number of building blocks of the original pattern.

In the activity that follows, we will practice creating a new pattern and then tell the computer to remove it from the model, so we will end up with a new model. We will change E to C:

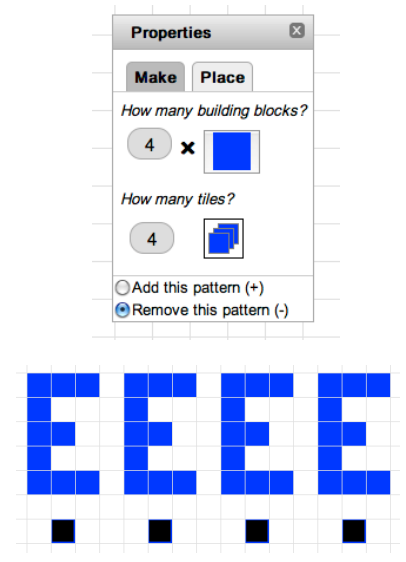
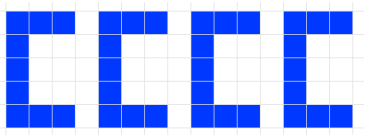
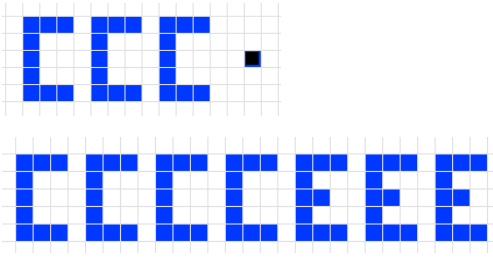
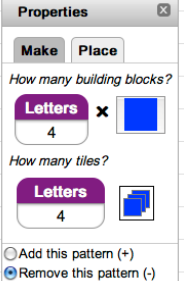

Theme – Building Negative Patterns

Activity 1 – Changing Letters

Title	Changing letters
eXpresser Objectives	Creating a new pattern and then removing it to create a new model
Mathematical Objectives	<ul style="list-style-type: none"> Introducing negative numbers
Teacher Notes	<p>Before leaving this activity, look carefully at the model rule in section 7:</p>  <p>Notice that it is showing a subtraction, but we have actually created a new pattern for our model! We turned it into a negative pattern when we turned it black, so in fact we added a negative pattern: 10 x Letters + -1 x Letters</p> <p>But the eXpresser has created: 10 x Letters - 1 x Letters</p> <p>This works because adding a negative number is the same as subtracting a positive number. Try it out for yourself!</p>



<p>1. To make the E into a C we need to remove the tile in the middle of the E, so we need to make a pattern that we can place over that tile.</p> <p>Create a building block of one blue tile. Place it below the E pattern.</p>	
<p>2. Now create a pattern from the blue building block, making sure it is placed exactly where the middle tile of all the E's in the model are.</p>	

<p>3. At the bottom of the Properties box for your new pattern, select “Remove this pattern”.</p> <p>This will turn your new pattern black.</p>	
<p>4. Now drag your new black pattern up to the middle tile of the E and drop it. What happens?</p>	
<p>5. Animate your model in the General Model. Are there E's and C's mixed up and sometimes black tiles like the examples shown in this figure?</p>	
<p>6. How can you make sure that for every E in your model there is one black tile?</p> <p>(You can do this by clicking and dragging the unlocked number on the slider to where you need it.)</p>	
<p>7. Give the model rule at the bottom of the eXpresser's working area.</p> <p>Is Smiley happy? Well done!</p>	

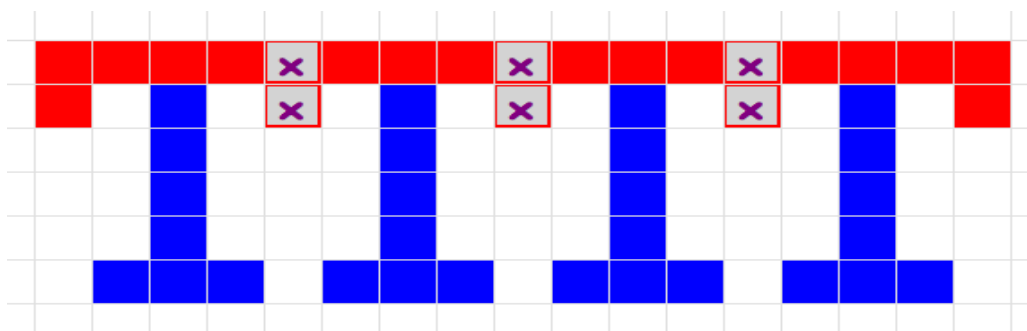
Theme – Overlapping Patterns

Activity 1 – Powerlines

Title	Powerlines
eXpresser Objectives	linking patterns, using negative patterns
Mathematical Objectives	<ul style="list-style-type: none"> • use of the additive inverse • use of negative patterns
Teacher Notes	<p>This activity follows from CHANGING LETTERS and develops the thinking behind subtracting parts of a pattern. It is presented as a model that needs fixing, so encourage students to work with the information that they have been given (by clicking on the model they will be able to see the properties of the powerlines model).</p> <p>The principle aim in this activity is to use negative patterns, by fixing up a pattern that has an overlap. A reminder of how to delete a tile in a pattern, either as a starter activity or once students have come up with a need to “get rid” of some tiles, may be useful. Students could redo the CHANGING LETTERS activity.</p>

Task / Activity

The principle aim in this activity is to use **negative patterns**, by fixing up a pattern that has an overlap. A reminder of how to delete a tile in a pattern, either as a starter activity or once students have come up with a need to “get rid” of some tiles, may be useful. Students could redo the [CHANGING LETTERS](#) activity.



Note that the model rule shows the addition of a “removed” pattern as a subtraction. This is discussed in the notes to the [CHANGING LETTERS](#) activity. These activities may be useful starting points for a discussion away from the eXpresser environment about the use of the additive inverse, as well as illustrating how adding a negative number has the same effect as subtracting a positive number. This may, for example, help with revision on operations on negative numbers.

$$7 \times \frac{\text{powerlines}}{4} + 7 \times \frac{\text{powerlines}}{4} - 2 \times \frac{\text{powerlines}}{4} - 1$$

Students will need to find a rule to say that there is one less overlap than the number of powerlines (i.e. $p-1$ times the number of tiles in the overlap). Operating on variables is an important, but difficult, algebraic skill that students need. (See the notes to the [LINES AND CROSSES](#) activity for more on this.)

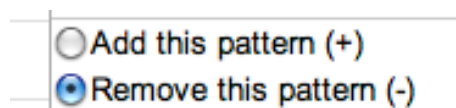
The aim of this activity is to use what you have learnt about adding and subtracting patterns to fix the model. Your task is to fix the model so that there are no overlapping powerlines.

To get started:

1. Remember you can only have **one** tile on a square. A cross on a tile means that there are too many tiles in that square. You will need to remove one.
2. How many patterns are the powerlines made with? Click on the powerlines to find the Properties box. This will give you some clues about how the powerlines are built.
3. Is there a pattern for where the crosses are? In the example below there is one cross **every four tiles**:



4. Think about how to make a pattern that matches your crosses. Use the slider to check your thinking about the pattern – when the slider is on 4, how many overlapping tiles are there? What about when it is on 6?
5. How does the pattern of overlapping tiles change when the slider does?
6. Use the “Remove this pattern” option to take away a pattern:



(If you need a reminder about how to do this, go back and look at the [CHANGING LETTERS](#) activity.)

7. Remember that you can create patterns on the working area away from your main pattern. This can give you room to work out how you need to build your patterns.

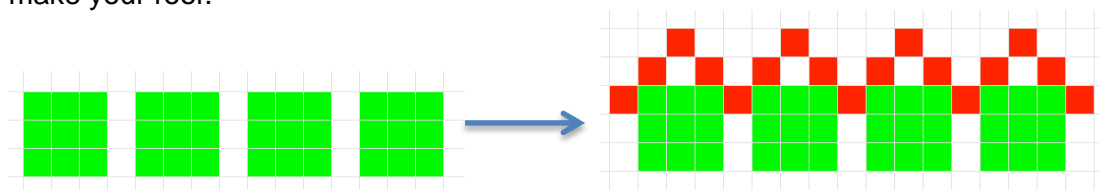
Activity 2 – Houses

Title	Houses
eXpresser Objectives	linking patterns, using negative tiling patterns
Mathematical Objectives	<ul style="list-style-type: none"> • confident use of constants and variables, • recognising equivalence of algebraic expressions, • use of the additive inverse, • how to use equivalent expressions
Teacher Notes	<p>One of the aims of this activity is to develop a familiarity with using negative patterns. Trials showed a general preference for additive approaches to model building, as opposed to using negative patterns. Perhaps this is because subtraction can be considered harder than addition, or that the use of negative patterns is less “obvious” than creating a model additively. The task can be completed without using a negative pattern, so, if use of a negative pattern is the teaching/learning focus of the activity, insist that the roofs be built (at least the first time) using a negative pattern.</p> <p>There are a number of ways the model can be built, and this is always helpful to reinforce the inverse nature of addition and subtraction, and “choosing” what suits a particular task. Allowing students to find and then compare their preferred view of the structure of the pattern is an important aspect of learning with the eXpresser because it develops the idea that structure can be seen differently, and exploring those differences requires a language of description.</p> <p>The eXpresser terminology is deliberately construction-based (“building blocks”, “make”, “place”, etc.). These, and the use of coloured tiles, allow discussion and comparison, and are all precursors to the more formal algebraic terminology architecture of terms, constants, brackets, co-efficients and expressions, etc.</p>

Task / Activity

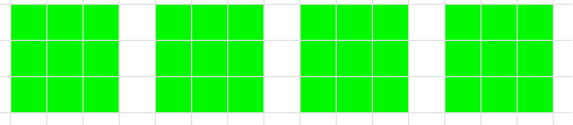
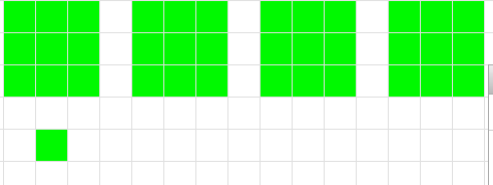
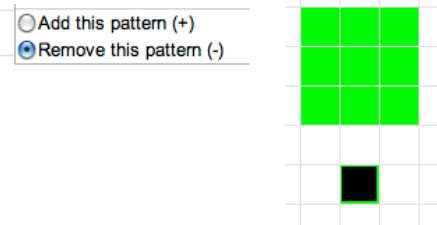
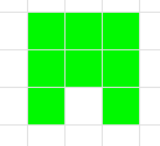
The aim of this activity is to link patterns. One pattern has been created for you, and your task is to build the required model by adding patterns. You will also need to think about how to take away parts of a pattern if there is an overlap.

You need to add a roof to the row of houses. You can use any colour or colours to make your roof.



Start students off on the structure of the roof – build one as a building block then see what happens when they make a pattern with it. It takes time to think through what is happening, so students may need some time in working out why they cannot just repeat the first roof, and how that might be able to be fixed using another pattern.

A reminder of how to delete a tile in a pattern, either as a starter activity or once students have come up with a need to “get rid” of some tiles, may be useful. One suggestion follows, with the students either watching or doing it themselves:

<p>1. Launch the activity.</p>	
<p>2. Before starting the roof exercise, suggest that we build a door first: create a one squared building block and pattern.</p>	
<p>3. Don't unlock any numbers; the intention here is to start the thinking.</p> <p>“Remove” the pattern.</p>	
<p>4. Drag the “negative door” pattern and drop it over the tile for the appropriate tile in the house.</p> <p>The result is a blank space.</p>	

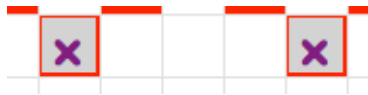
This exercise provides a reminder of negative patterns. We have created a pattern specifically to delete one tile in the 3x3 square building block (i.e. $1 + -1 = 0$), which produced a door in the house. In the task, a pattern needs to be created to remove the overlap, so that the tile between the houses is coloured and not marked with a cross (i.e. $2 + -1 = 1$). This may need to be made explicit.

To get started:

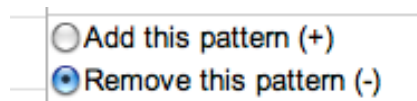
1. Start by building the roof for the first house. Click on the house to get the Properties box. This will give you some clues about how the house pattern is built. See what you can use for the roof pattern.
2. Remember that you can create patterns on the working area away from your main pattern. This can give you room to work out how you need to build your model.

- You can only have one tile on a square. If you get a square on your canvas that has a cross on it, that means there are too many tiles in that square. You will need to remove it (or them).
- Is there a pattern to when you get a cross?

Here it happens every 4 tiles:



- Think about how to make a pattern that matches your crosses and then taking it away using the “Remove this pattern” option:



(If you need a reminder about how to do this, go back to the [CHANGING LETTERS](#))

Two solutions to the activity are:

	$\frac{\text{houses}}{4} \times 9 + \frac{\text{houses}}{4} \times 5 - \frac{\text{houses}}{4} - 1$
	$\frac{\text{houses}}{4} \times 9 + \frac{\text{houses}}{4} \times 4 + 1$

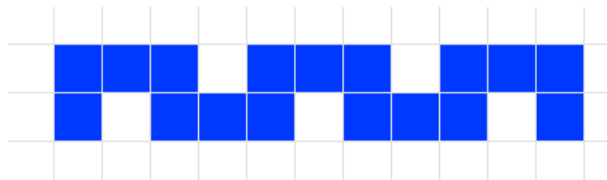
Some further points that may be taken up are set out below:

- The model rules show how the model is built– each term in each of the rules refers to a colour in the model. Can students identify which term goes with which colour? Get students to “tell the story” of each solution from the model rule – this provides a rationale for using brackets, and is a good starting point for creating expressions.
- How does the model rule show the instruction to “take away one less than the number of houses” in the first solution? (Another way of getting to the same solution is take away as many as the number of houses and then add one back; this illustrates that subtracting a negative number has the same effect as adding a positive number, and why we have the rule about two minus’s making a plus.
- The first and second solutions look similar – ask the class what is the same and what is different about them? The differences in each of the model rules reflect a different construction. Colour has been used here to show how the different models have been created. You might want to refer to the rules in formal algebraic notation and ask students to justify their equivalence.

Theme – Linking Unlocked Numbers

Activity 1 – Help Tim

Title	Help Tim
eXpresser Objectives	Solving the problem of two unlocked numbers
Mathematical Objectives	<ul style="list-style-type: none"> • Creating and linking variables
Teacher Notes	<p>This activity is presented with two patterns, each with an unlocked number. If the sliders are set manually then the model appears to work, i.e. with an “arch” at the beginning and the end. Students often unlock several numbers and struggle with how to connect the number of one building block, or pattern with another. This activity is designed to make this connection explicit by considering how the number of “arches” and “links” is related. This is done by examining the changes to each of the patterns that make up the model, and then looking at how the number of one pattern is linked to the other.</p>








In this activity, you need to help Tim. He has used two patterns to create this model. The problem is that when he animates in the General Model, it messes up and it does not work the way he wants it to, with a column at the beginning and end:



1. Launch the activity.
2. Animate the pattern in the General Model and look at what is happening. Write a sentence to describe what you notice:

3. Now use the sliders. What happens if both sliders are set at 3? What if they are both set at 2? Write a sentence to describe HOW the model is not correct.

4. Write in the table below the setting for each slider that is needed to make the model correct:

Model		
		
		
		

5. Decide which of these statements is correct:





Look at the MODEL:	True or false
There is a link between each arch .	
There is an arch between each link .	
Look at the TABLE or THE SLIDER:	
The number of links is one less than the number of arches .	
The number of arches is one less than the number of links .	
The number of links is one more than the number of arches .	
The number of arches is one more than the number of links .	

6. How would you describe the connection between the values of the sliders when they are set so that the model works?

7. What is the rule for the number of tiles in the arch pattern (Look at the Properties box)?


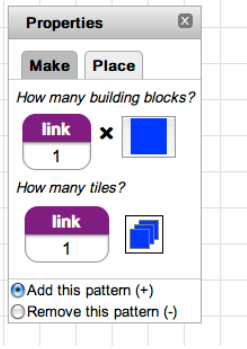
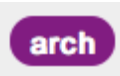

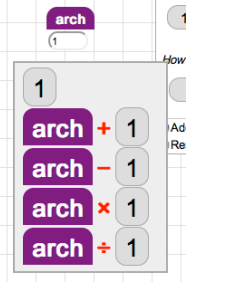
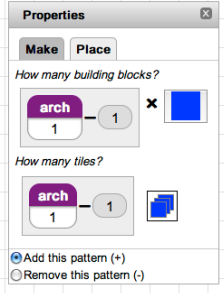
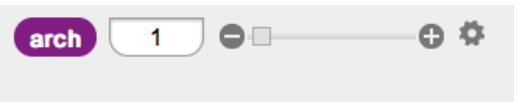
8. What is the rule for the number of tiles in the link pattern?

9. Now, circle the correct statement:

  = + 1	  = - 1
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Check your answer by setting the arch slider to 6. How many links do you need? 7 or 5? Does this fit your rule?

10. We only need one slider, because in this model the number of links will always be one more than the number of arches. Follow the steps below:

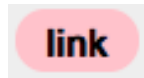
STEP	ACTION
<p>Left click in the Properties box of the Link pattern to relock it.</p> <div style="text-align: center; margin-top: 10px;">  </div>	
<p>Create an expression for the number of links by dragging 'arch' and '1' to the canvas.</p> <div style="text-align: center; margin-top: 10px;">   : </div>	
<p>Drag your expression to the Properties box:</p>	
<p>Set the slider to 1. What is the smallest value that 'arch' can take for the model to work? _____</p> <p>Click on the settings button and adjust the limits.</p>	

11. Now animate the General Model. Have you fixed Tim's problem? Does the model start and finish with a vertical column?



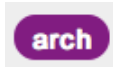
12. Finish the activity by giving the Model Rule (make sure Smiley is happy).

Write your Model Rule here:

EXTENSION: Can you find the model rule using



?

(Hint: if  =  - 1, then  =??)

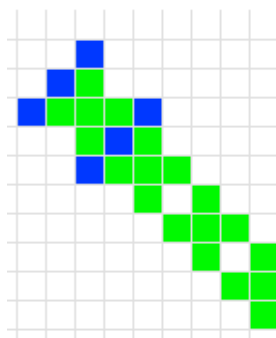
Activity 2 – Lines and Crosses

Title	Lines and Crosses
eXpresser Objectives	linking patterns, unlocking numbers
Mathematical Objectives	<ul style="list-style-type: none">• operations on variables,• use constant and variable terms,• recognise equivalence of algebraic expressions
Teacher Notes	<p>This activity can be done as a standalone activity, or as a follow on from CROSSES. CROSSES looks at placement of patterns and how to vary the horizontal placement that the eXpresser defaults to. This exercise is designed to encourage students to link the use of variables, and to consider how one pattern varies with another. It is useful to do the activity beforehand, perhaps with colleagues, to see what structure or structures you “see” and work with, as there are a number of approaches that can be made.</p> <p>The diagonal positioning of this pattern presents an initial challenge (and the CROSSES activity deals with this aspect explicitly), as the horizontal/vertical placement needs to be varied. Students will need time to try various combinations. If they get stuck, consider prompting them to look carefully at the “Place” tab in the properties box of the crosses pattern, and use that information.</p> <p>There are some notes below about suggestions on how you might wish to support students’ understanding of unlocked numbers, eXpresser’s metaphor on variables.</p> <p>Students often unlock a second number for the blue lines. This is a logical step, and follows the idea implicit in the name of the unlocked number in the crosses pattern (“number of crosses”). Reflecting on what happens when this is done is useful, as is noticing eXpresser’s feedback system, and how the model will “mess up”. The screenshot below shows a second unlocked number called “no of lines”. Expressing the number of lines using the number of crosses is not intuitive, and takes time to become apparent.</p>

Task /Activity



If/Once a second number is unlocked, a rule can be given to the blue pattern to get it coloured (remember that colouring a pattern is part of the feedback system – colour means that the rule is correct for that specific value given to the unlocked number). Encourage the use of the sliders to explore when the model “works” and remains coloured. Students may think they have completed the task when they find one instance that “works”, i.e. when there is a blue line at either end of the model. Animating the General Model will show that the activity is not complete, because the pattern “messes up” when the General Model is animated:

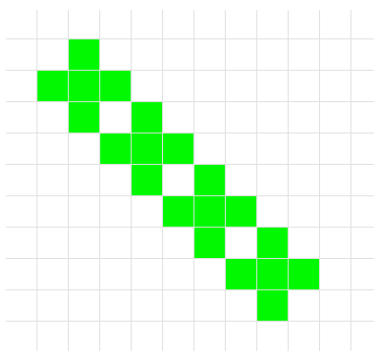


This means they have found that their solution holds for a specific instance, and now they need to work on the general case: what other instances can they find when the model “behaves”? Encourage them to “say what they see”, so that the specific (such as “3 lines and 2 crosses”, “6 lines and 5 crosses”) can move to the general (“one more line than crosses”, which we express mathematically as \dots “crosses +1”). Treating a variable like a number and performing operations on it, such as adding 1, has been identified in the research as a difficult step for students. It can take a long time and much questioning and discussion.

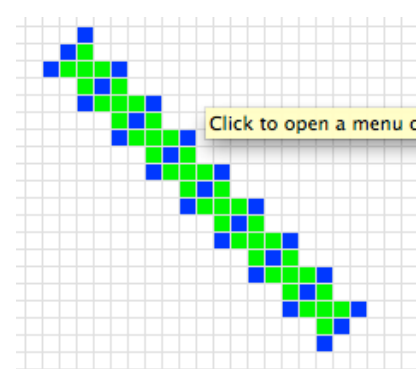
Once the link between the two patterns is established, students who have unlocked more numbers will need to lock them again. This is covered step by step in the [HELP TIM](#) activity. They will then need to derive the Model Rule.

The aim of this activity is to link patterns. Your task is to build the model by creating the pattern that is missing. You will need to think carefully about how patterns can grow or ‘shrink’ together, and to think about how to use one pattern to describe the way another pattern changes.

Your task is to turn this:



into this:



To get started:

1. Click on the crosses pattern to find the properties box. This has lots of clues that can help you with your pattern. Use the "Place" tab too.
2. Think about how the pattern of blue tiles is connected to the green pattern one. How many blue lines are there for each cross?

Remember to give your model rule (for both the lines and crosses) once you have made your model. Then you'll get your big green smile!

Theme – Changing the Translation

Activity 1 - Crosses

Title	Crosses
eXpresser Objectives	Understand pattern placement and unlocking numbers
Mathematical Objectives	<ul style="list-style-type: none">• translation vectors,• diagonal vs. horizontal/vertical placements,• describing translation/position
Teacher Notes	<p>When a pattern is created, the eXpresser defaults to a horizontal placement with four repetitions. The diagonal positioning of this pattern presents the challenge, as the horizontal/vertical placement needs to be varied. Students will need time to try various combinations, and it is helpful to provide ways to encourage them to describe what is happening when they change the horizontal/vertical components of the vector.</p> <p>For example, note that the default horizontal placement will always place the pattern so that there is just one tile between the end of one building block and the beginning of the next building block. The number in the horizontal placement vector, however, will show the number of squares in the working area needed to translate one tile in a building block to its next appearance in the pattern to achieve that placement.</p> <p>This can be drawn out in discussion initially by asking how far apart the building blocks are placed – answers like “with a gap of 1 tile” or “1 tile apart”, while correct, will need more elicitation as this describes where the building blocks are, but that will not allow the eXpresser to “know” where to place each tile in the pattern – to do this, we need to give an instruction about how far apart each tile is repeated. This is part of the movement from observing and describing the model to analysing the structure of the model in a mathematical sense. Answers such as “leave a gap of 5 tiles” need to be rethought, and then rephrased as “repeat every _____ tiles”.</p> <p>A number can be typed directly into the box once the pattern is created, but after the pattern has been created, the number generator needs to be used to change the placement components.</p> <p>(Note that only whole numbers or zero can be used)</p> <p>Set out below are a few thoughts about some other mathematical concepts that this activity could be used for class discussions.</p>

Describing position:

A preliminary discussion of the ways we describe location or placement might be useful to introduce this activity. For example, remind (or explain, depending upon prior knowledge) students that the (x, y) co-ordinates of the Cartesian plane allows us to describe position, and translation vectors are used to describe changes in position and the eXpresser is similar, but different because:

In the eXpresser, the positive direction is downward

Negative placement:

Encourage students to try different numbers: what happens if zero is used in either component of the placement vector? What about a negative number? How would you get a vertical pattern?

Using negative numbers to describe directional differences is an important aspect of “directed” numbers, and students will come across it with translations. It is, however, often not always made explicit in texts, where the focus may be more on operations on negative numbers.

The Reindeer activity requires students to change the direction of one pattern in a model and is a useful activity to set students after [CROSSES](#), perhaps as a homework task.

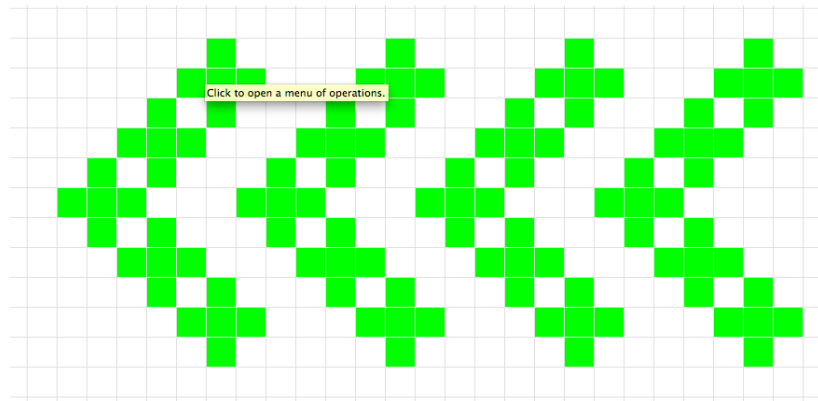
Zero:

Using negative numbers in this way also highlights the role of zero as the origin, or placeholder, neither one direction nor the other. This could be used to encourage a discussion about how zero does not act like other numbers (for example, we can't divide by it, we get zero when we multiply by it, we get 1 when we raise any number to the power of zero, etc.).

Unlocking:

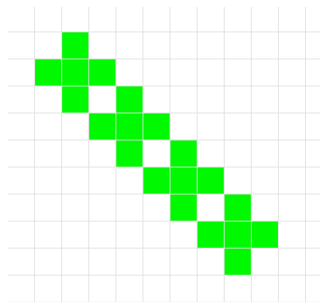
It may be necessary to remind students that animating in the General Model will show the effect of unlocking a number (that is, the General Model allows them to “keep an eye on the general” because it assigns random values to the unlocked number) while looking at the particular, which is a fundamental design feature of the eXpresser. When students complete the activity, they can be challenged to create a model that animates in the opposite direction (i.e. upwards and to the left), using negative numbers for the components.

The model below (which shows the 4th term) has been created by copying the finished model (using “Copy” from the menu of operations) and changing some of the placement components. Students can be challenged to create it:



Note that [LINES AND CROSSES](#) has a model of Crosses embedded, with the task being to create lines in between. Students may be directed to this activity after completing Crosses, and use the Properties of the given Crosses model to check how it has been done, and confirm that their own model is correct.

The aim of this activity is to create a model that animates in a different direction to earlier activities. The task is to create a line of crosses like this:



Think about how to place the pattern after you make a building block (remember to look at the “[Place]” tab) so that the second cross just touches the first, and the third one just touches the second, and so on.